03/05/2013 PHY 306: Midterm 1

Name

This exam is closed book. You are allowed an index card not larger than 4"x6" (both sides writing allowed) with information of your own choosing. The exam period is 10:00 AM-11:20 AM.

Some numerical constants:

k_B=Boltzmann constant= 1.38x10^{-23} J/K
N_A=Avogadro’s number=6.02x10^{23}
R=gas constant=8.315 J/mol-K
1atm=1.013 x10^{5} N/m^{2}

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Problem 1 (30 Points)
A system is comprised of 0.25 mole of an ideal H₂ gas at 0°C and P=1 atm. When 3400 J of thermal energy are added to the system at constant pressure, the resultant expansion causes the system to perform 900 J of work. Calculate:

(a) (5 points) The initial state \( (P, V, T) \)

\[
P_i = 1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2
\]

\[
T_i = 273 K
\]

\[
V_i = \frac{nRT_i}{P_i} = 0.0056 \text{ m}^3
\]

(b) (5 points) The final state \( (P, V, T) \)

\[
P_F = P_i
\]

\[
W = -P \Delta V = \int_{V_i}^{V_F} -PdV = -P(V_F - V_i) = -900 J
\]

\[
T_F = \frac{P_F V_F}{nR} = 706.6 K
\]

\[
\Delta V \approx 0 \Rightarrow \Delta V = 0.0056 \frac{1.013 \times 10^5 \text{ N}}{\text{m}^2} = 0.0089 \text{ m}^3
\]

\[
V_F = 0.0056 + 0.0089 = 0.0145 \text{ m}^3
\]
(c) (10 points) The change in internal energy for the process, $\Delta U$

$$
\Delta U = Q + W = \text{2500 J}
$$

(d) (10 points) The change in entropy for the process, $\Delta S$.

$$
\Delta S = nR \ln \left( \frac{V_f}{V_i} \right) + \frac{5}{2} nR \ln \left( \frac{T_f}{T_i} \right) = 0.25 \text{ mol} \times 8.315 \text{ J} / \text{mol} \text{ K} \left[ \ln \left( \frac{19.5}{5.6} \right) + \frac{5}{2} \ln \left( \frac{706}{273} \right) \right] = 6.925 \text{ J} / \text{K}
$$
Problem 2 (35 points)
You have an Einstein solid with three oscillators and a two-state paramagnet with four spins. The magnetic field in the region of the paramagnet points "up" and is carefully tuned so that $\mu B = \varepsilon$, where $\mu B$ is the energy of a spin pointing "down", $-\mu B$ is the energy of a spin pointing "up", and $\varepsilon$ is the energy level separation of the oscillators. At the beginning of the experiment the energy in the Einstein solid $U_s$ is $4 \varepsilon$ and the energy in the paramagnet $U_p$ is $-4 \varepsilon$.

(a) (5 points) Using a schematic drawing of the Einstein solid, give an example of a microstate which corresponds to the macrostate $U_s = 4 \varepsilon$.

(b) (5 points) Using a schematic drawing of the paramagnet, give an example of a microstate which corresponds to the macrostate $U_p = -4 \varepsilon$. 
(c) (10 points) Considering that the “system” comprises the solid and the paramagnet, calculate the multiplicity of the system assuming that the solid and paramagnet cannot exchange energy.

\[ \Omega_{sp} = \Omega_s - \Omega_p \]

No energy exchange \( \Rightarrow \Omega_p = 1 \) (only 1 state with \( N = 4 \epsilon \))

\[ \Omega_s (\epsilon = 4, N = 3) = \frac{4!}{2!1!} \frac{1}{2!} = \frac{24}{4} = 6 \]

\[ \Omega_{sp} = 15 \]

(d) (15) Now let the solid and paramagnet exchange energy until they come to thermal equilibrium. Note that because this system is small, there will be large fluctuations around thermal equilibrium, but let’s assume that the system is not fluctuating at the moment.

What is the value of \( U_s \) now? Draw an example of a microstate in which you might find the solid.

What is the value of \( U_p \) now? Draw an example of a microstate in which you might find the paramagnet.

Note \( \Delta \epsilon = 2 \epsilon \) (minimum energy we need to excite 1 spin)

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<tr>
<th>( U_s )</th>
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<th>( U_p )</th>
<th>( \Omega_p )</th>
<th>( \Omega_{sp} )</th>
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<tr>
<td>4( \epsilon )</td>
<td>15</td>
<td>-4( \epsilon )</td>
<td>1</td>
<td>15</td>
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<tr>
<td>2( \epsilon )</td>
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<td>( \frac{4!}{3!1!} = 4 )</td>
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Max \( \Omega_{sp} \)

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<th>( \frac{N!}{(N-M)!M!} )</th>
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\( \frac{N!}{(N-M)!M!} \) states of solid

\( \frac{1}{2} \) state of \( p \)

State of solid

State of \( p \)
Problem 3 (35 points)

One mole of a monoatomic ideal gas goes through a quasistatic three stage cycle (1-2,2-3,3-1) as shown in the figure in the left. Process 3-1 is adiabatic. Assume that the volumes V1 and V2 and the Pressure P1 are given, therefore all the answers should be given in terms of these three variables.
(a) (15 points) Calculate the work $W$ done on/by the gas during each step of the cycle. Add your Answers to the table above. Make sure you indicate whether the work is positive, negative or 0. Credit will be given if you only get the sign right even if you do not get the formula right.

1-2 $P=\text{constant} = P_1$

\[ W = -\int_{V_1}^{V_2} P_1 \, dV = -\frac{P_1(V_2 - V_1)}{V_1} \leq 0 \]

2-3 $V=\text{constant} = V_2 \implies \Delta V = 0 \implies W = 0$

3-1 $\gamma$-adiabatic $PV = \text{constant} = P_1 V_1^{\gamma} \implies \frac{P V^{\gamma}}{V_1^{\gamma}} = P V^{\gamma} = \frac{P_1 V_1^{\gamma}}{V_1^{\gamma}} \implies P = \frac{P_1 V_1^{\gamma} V_2^{\gamma-1}}{V_2^{\gamma}}$

\[ W = -\int_{V_2}^{V_1} P \, dV = \int_{V_2}^{V_1} \frac{P_1 V_1^{\gamma} V_2^{\gamma-1}}{V_2^{\gamma-1}} \, \frac{V_2^{\gamma-1}}{V_2^{\gamma}} \, V^{\gamma-1} \, V^{-\gamma-1} \, dV = \frac{P_1 V_1^{\gamma}}{\gamma - 1} \left( \frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}} \right) \]

$\gamma = \frac{5}{3} > 1$
(b) (15 points) Calculate the entropy change $\Delta S$ for each stage (1-2,2-3,3-1) of the cycle, each step of the cycle. Add your Answers to the table above. Make sure you indicate whether the change is positive, negative or 0. Credit will be given if you only get the sign right even if you do not get the formula right.

$$\Delta S = R \ln \frac{V_f}{V_i} + \frac{3}{2} R \ln \frac{T_f}{T_i} = R \left( \ln \frac{V_f}{V_i} + \frac{3}{2} \ln \frac{T_f}{T_i} \right)$$

1-2 $T_1$ constant $P_1$, $V_i = V_1$, $V_f = V_2$

$$T_1 = \frac{P_1 V_1}{nR}, \quad T_2 = \frac{P_2 V_2}{nR} \quad \Rightarrow \Delta S = R \left( \ln \frac{V_2}{V_1} + \frac{3}{2} \ln \frac{V_2}{V_1} \right) - \frac{5}{2} \ln \left( \frac{V_2}{V_1} \right)$$

2-3 $V = V_2$ constant $\ln \frac{V_f}{V_i} = 0$

$$T_3 = \frac{P_3 V_3}{nR}, \quad T_4 = ? = \frac{P_4 V_4}{nR} \quad \Rightarrow P_2 \text{ is not a known constant}$$

Obtain $T_2$ from the adiabatic: $P_2 V_2^\gamma = P_1 V_1^\gamma \Rightarrow P_2 = P_1 \left( \frac{V_1}{V_2} \right)^\gamma$

$$\Delta S = R \ln \left( \frac{T_f}{T_i} \right) = R \ln \left( \frac{P_1 \left( \frac{V_1}{V_2} \right)^\gamma V_2}{P_1 V_1^\gamma} \right) = R \ln \left( \frac{V_1}{V_2} \right) = \frac{5}{2} R \ln \left( \frac{V_1}{V_2} \right)$$

3-1 Quasistatic + adiabatic = isentropic $\Delta S = 0$

(c) (5 points) Is this full cycle reversible? Why or Why not?

Yes, The Total $\Delta S = 0$
Page for extra calculations (or extra space needed for answers)