Preparation for the Final Exam

• There will be 5 problems, 1 from 1st half, 1 from 2nd half and 3 from last part

• For first 2 problems: Content will be identical to first two midterms. If something was not on the midterm it will not be on the final. But this does not mean that the problems will be identical to the midterm problems. Will be similar, but by no means identical

• For the last 3 problems: I will follow this review and the homework you just handed in.

• What will be included: Full Chapter 6, Chapter 7 only up to 7.3 (included)
Problem: Partition function

1.- Consider a system of distinguishable particles with five microstates with energies 0, $\varepsilon, \varepsilon, \varepsilon$ and 2$\varepsilon$ ($\varepsilon=1$ eV) in equilibrium with a reservoir at temperature $T=6000$ K.

1.- Find The partition Function of the System
2.- Find the mean Energy of the system
3.- What is the energy of a system with $N=10$ of these particles?
2.- Consider a system of N particles with only 3 possible energy levels separated by $\varepsilon$ (let the ground state energy be 0). The system occupies a fixed volume $V$ and is in thermal equilibrium with a reservoir at temperature $T$. Ignore interactions between particles and assume that Boltzmann statistics applies.

(a) What is the partition function for a single particle in the system?
(b) What is the average energy per particle?
(c) What is probability that the $2\varepsilon$ level is occupied in the high temperature limit, $k_B T >> \varepsilon$? Explain your answer on physical grounds.
(d) What is the average energy per particle in the high temperature limit, $k_B T >> \varepsilon$?
(e) At what temperature is the ground state 1.1 times as likely to be occupied as the $2\varepsilon$ level?
(f) Find the heat capacity of the system, $CV$, analyze the low-$T (k_B T << \varepsilon)$ and high-$T (k_B T >> \varepsilon)$ limits, and sketch $CV$ as a function of $T$. 
Problem: Boltzman Distribution

3.- Consider an ideal gas of atoms with mass $m$ at temperature $T$.
(a) Using the Maxwell-Boltzmann distribution for the speed $v$, find the corresponding distribution for the kinetic energy $\varepsilon$ (don’t forget to transform $dv$ into $d\varepsilon$).
(b) Find the most probable value of the kinetic energy.
(c) Does this value of energy correspond to the most probable value of speed?
4.- When the copper atoms form a crystal lattice with the density of atoms of $n=8.5 \cdot 10^{28} \text{m}^{-3}$, each atom donates 1 electron in the conduction band.

(a) Assuming that the effective mass of the conduction electrons is the same as the free electron mass, calculate the Fermi energy. Express your answer in eV.

(b) The electrons participate in the current flow if their energies correspond to the occupancy $n(\varepsilon)$ that is not too close to 1 (no empty states available for the accelerated electrons) and not too small (no electrons to accelerate). At $T=300 \text{K}$, calculate the energy interval that is occupied by the electrons that participate in the current flow, assuming that for these electrons the occupancy varies between 0.1 and 0.9.

(c) Using the assumptions of (b), calculate the ratio $N_1/N_0$ where $N_1$ is the number of “current-carrying” electrons, $N_0$ is the total number of electrons in the conduction band. Assume that within the range where the occupancy varies between 0.1 and 0.9, the occupancy varies linearly with energy (see the Figure), and the density of states is almost energy-independent. The density of states for the three-dimensional Fermi gas:

\[ g(\varepsilon) = \frac{3N}{2E_F^{3/2}} \sqrt{\varepsilon} \]
5.- For a system of particles at room temperature,
(a) how large must \( \varepsilon - \mu \) be for the Fermi-Dirac, Boltzmann, and Bose-Einstein distributions agree within 1%?
(b) Estimate the density of a system of mobile electrons in a semiconductor that can be treated at room temperature equally well (with 1% accuracy) using all three distributions. Assume that the effective electron mass is the same as a free electron mass, and that you can use for this estimate the expression for \( \mu \) in an ideal classical gas.
6. Consider a mixture of Hydrogen and Helium at T=300 K. Find the speed at which the Maxwell distributions for these gases have the same value.